splinart Documentation

Loic Gouarin

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CONTENTS

splinart is a package used for a tutorial which explains how to do the Python packaging using

- PyPi
- conda build
- pytest
- Pylint
- Sphinx

And automate the process to distribute this package using github.

The original idea of splinart is found on the great invonvergent website.

If you want to install splinart:

```
pip install splpinart
```

or:

```
conda install -c gouarin splinart
```

CHAPTER

ONE

USER MANUAL

1.1 Cubic Spline

We consider here a cubic spline passing through the points (x_i, y_i) with $a = x_1 < \ldots < x_n = b$, that is, a class function $\mathcal{C}^2$ on $[a, b]$ and each restriction at the interval $[x_{i-1}, x_i]$, $1 \leq i \leq n$, is a polynomial of degree less than 3. We will note S such a spline. His equation is given by

$$S_i(x) = Ay_i + By_{i+1} + Cy''_i + Dy''_{i+1}, \qquad x_i \leq x \leq x_{i+1},$$

where

$$A = \frac{x_{i+1} - x}{x_{i+1} - x_i} \qquad \text{et} \qquad B = \frac{x - x_i}{x_{i+1} - x_i},$$

$$C = \frac{1}{6}\left(A^3 - A\right)\left(x_{i+1} - x_i\right)^2 \qquad \text{et} \qquad D = \frac{1}{6}\left(B^3 - B\right)\left(x_{i+1} - x_i\right)^2.$$

If we derive this equation twice with respect to x, we get

$$\frac{d^2S(x)}{dx} = Ay''_i + By''_{i+1}.$$

Since $A = 1$ in x_i and $A = 0$ in x_{i+1} and conversely for B, we can see that the second derivative is continuous at the interface of the two intervals $[x_{i-1}, x_i]$ and $[x_i, x_{i+1}]$.

It remains to determine the expression of y''_i. To do this, we will calculate the first derivative and impose that it is continuous at the interface of two intervals. The first derivative is given by

$$\frac{dy}{dx} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} - \frac{3A^2 - 1}{6}(x_{i+1} - x_i)y''_i + \frac{3B^2 - 1}{6}(x_{i+1} - x_i)y''_{i+1}.$$

We therefore want the value of the first derivative in $x = x_i$ over the interval $[x_{i-1}, x_i]$ to be equal to the value of the first derivative in $x = x_i$ over the interval $[x_i, x_{i+1}]$; which gives us for $i = 2, \ldots, n-1$

$$a_i y''_{i-1} + b_i y''_i + c_i y''_{i+1} = d_i,$$

with

$$\begin{aligned}
a_i &= \tfrac{x_i - x_{i-1}}{x_{i+1} - x_{i-1}} \\
b_i &= 2 \\
c_i &= \tfrac{x_{i+1} - x_i}{x_{i+1} - x_{i-1}} \\
d_i &= \tfrac{6}{x_{i+1} - x_{i-1}} \left(\tfrac{y_{i+1} - y_i}{x_{i+1} - x_i} - \tfrac{y_i - y_{i-1}}{x_i - x_{i-1}} \right).
\end{aligned}$$

So we have $n-2$ linear equations to calculate the n unknowns y_i'' for $i = 1, \ldots, n$. So we have to make a choice for the first and last values and we will take them equal to zero. We can recognize the resolution of a system with a tridiagonal matrix. It is then easy to solve it by using the algorithm of Thomas which one recalls the principle

$$c_i' = \begin{cases} \frac{ci}{b_i} & i = 1 \\ \frac{c_i}{b_i - a_i c_{i-1}'} & i = 2, \ldots, n. \end{cases}$$

$$d_i' = \begin{cases} \frac{di}{b_i} & i = 1 \\ \frac{d_i - a_i d_{i-1}'}{b_i - a_i c_{i-1}'} & i = 2, \ldots, n. \end{cases}$$

The solution is then obtained by the formula

$$\begin{array}{l} y_n'' = d_n' \\ y_i'' = d_i' - c_i' y_{i+1}'' \qquad \text{pour} \qquad i = n-1, \ldots, 1. \end{array}$$

CHAPTER

TWO

TUTORIAL

2.1 Splinart on a circle

In this tutorial, we will see how to use splinart with a circle.

First of all, we have to create a circle.

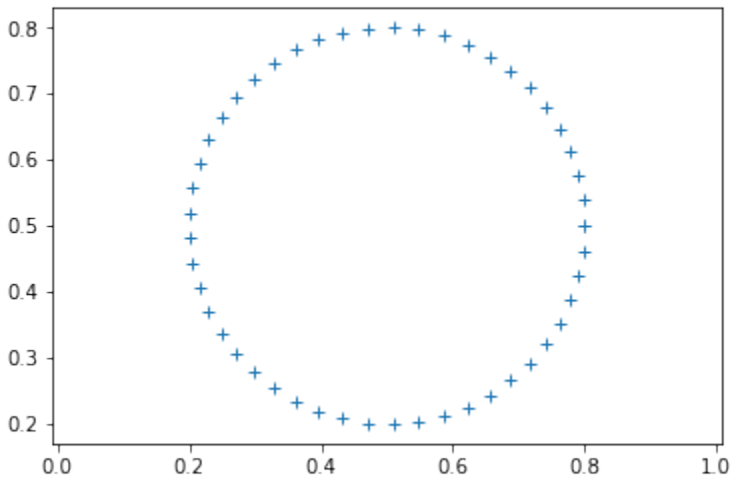
[1]:
```
import splinart as spl
center = [.5, .5]
radius = .3
theta, path = spl.circle(center, radius)
```

In the previous code, we create a discretization of a circle centered in $[0.5, 0.5]$ with a radius of 0.3. We don't specify the number of discretization points. The default is 30 points.

We can plot the points using matplotlib.

[2]:
```
%matplotlib inline
```

[3]:
```
import matplotlib.pyplot as plt
plt.axis("equal")
plt.plot(path[:, 0], path[:, 1], '+');
```



2.1.1 The sample

In order to compute a sample on a given cubic spline equation, we need to provide a Python function that gives us the x coordinates. We can choose for example.

[4]:
```
import numpy as np
def x_func():
    nsamples = 500
    return (np.random.random() + 2 * np.pi * np.linspace(0, 1, nsamples))%(2*np.pi)
```

We can see that the points are chosen between $[0, 2\pi]$ in a random fashion.

2.1.2 The cubic spline

Given a path, we can apply the spline function in order to compute the second derivative of this cubic spline.

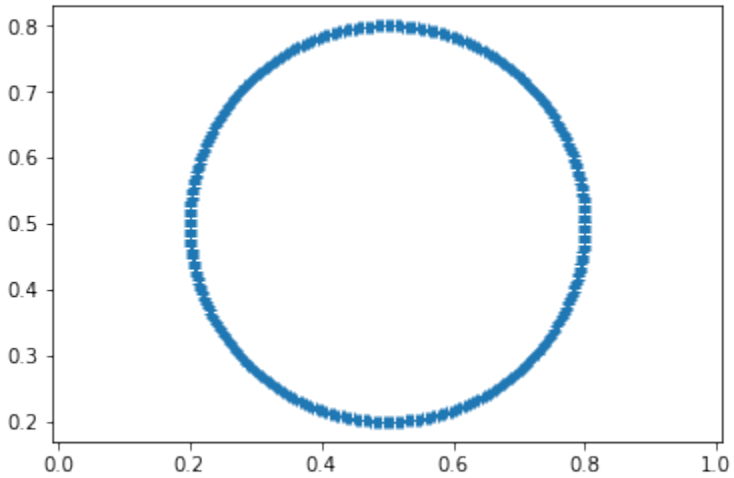
[5]:
```
yder2 = spl.spline.spline(theta, path)
```

And apply the equation to the sample

[6]:
```
xsample = x_func()
ysample = np.zeros((xsample.size, 2))
spl.spline.splint(theta, path, yder2, xsample, ysample)
```

which gives

[7]:
```
import matplotlib.pyplot as plt
plt.axis("equal")
plt.plot(ysample[:, 0], ysample[:, 1], '+');
```

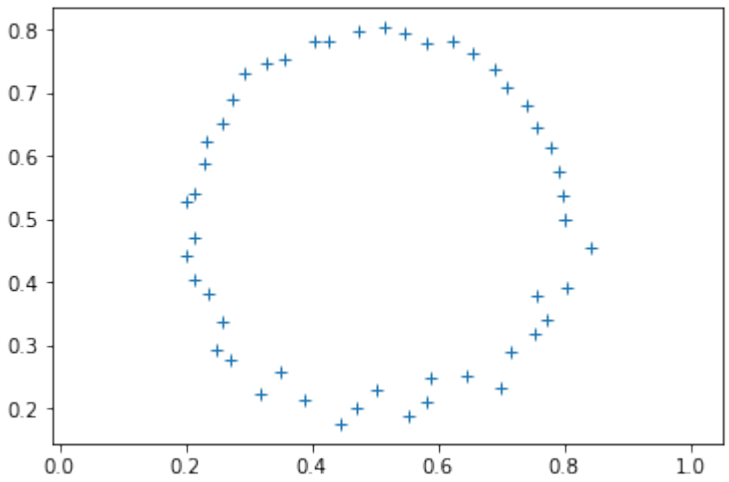


We can see the sample is well defined around the circle that we defined previously.

Now, assume that we move randomly the points of the circle with a small distance.

[8]:
```
spl.compute.update_path(path, scale_value=.001, periodic=True)
```

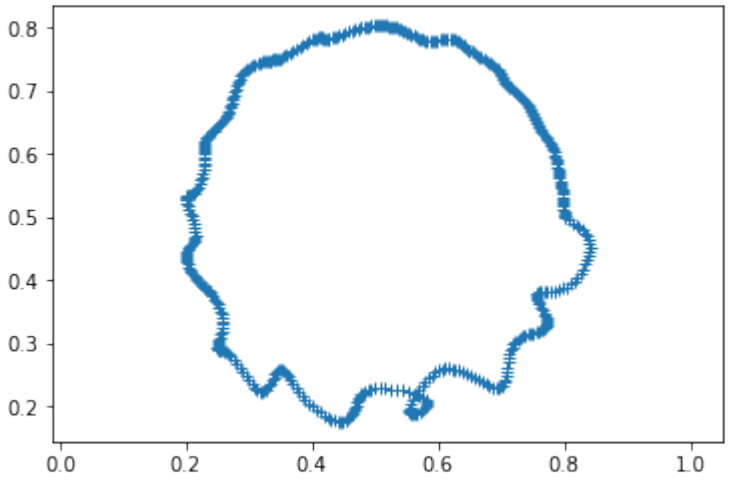
[9]:
```
import matplotlib.pyplot as plt
plt.axis("equal")
plt.plot(path[:, 0], path[:, 1], '+');
```



And we compute again the sample of the new cubic spline equation.

[10]:
```
yder2 = spl.spline.spline(theta, path)
spl.spline.splint(theta, path, yder2, xsample, ysample)
spl.compute.update_path(path, scale_value=.001, periodic=True)
```

[11]:
```
import matplotlib.pyplot as plt
plt.axis("equal")
plt.plot(ysample[:, 0], ysample[:, 1], '+');
```



The circle is deformed.

This is exactly how works splinart. We give a shape and at each step

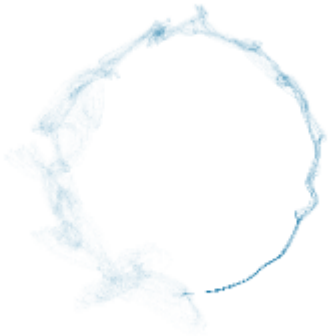
- we perturb the points of this shape
- we compute a sample an this new cubic spline equation
- we add the pixel with a given color on the output image

And we do that several time. We can have the following result

[12]:
```
img_size, channels = 1000, 4
img = np.ones((img_size, img_size, channels), dtype=np.float32)

theta, path = spl.circle(center, radius)
spl.update_img(img, path, x_func, nrep=4000, x=theta, scale_value=.00005)
```

[13]:
```
spl.show_img(img)
```



[ ]:

CHAPTER

THREE

REFERENCE MANUAL

3.1 splinart

3.1.1 splinart package

Subpackages

splinart.shapes package

Submodules

splinart.shapes.base module

Define basic shapes.

splinart.shapes.base.**circle**(*center*, *radius*, *npoints=50*)
Discretization of a circle.

Parameters

- **center** (*list(2)*) – 2d coordinates of the center.
- **radius** (*float*) – Radius of the circle.
- **npoints** (*int*) – Number of discretization points (the default value is 50).

Returns

- *np.ndarray* – The theta angle.
- *np.ndarray* – The 2d coordinates of the circle.

splinart.shapes.base.**line**(*begin*, *end*, *ypos=0.5*, *npoints=50*)
Discretization of a horizontal line.

Parameters

- **begin** (*float*) – The left point of the line.
- **end** (*float*) – The right point of the line.
- **ypos** (*float*) – The position of the y coordinate (the default value is 0.5).
- **npoints** (*int*) – Number of discretization points (the default value is 50).

Returns The 2d coordinates of the line.

Return type np.ndarray

Module contents

Shape package

splinart.spline package

Submodules

splinart.spline.spline module

Cubic spline

splinart.spline.spline.**spline**(*xs*, *ys*)
Return the second derivative of a cubic spline.

Parameters

- **xs** (*np.ndarray*) – The x coordinate of the cubic spline.
- **ys** (*np.ndarray*) – The y coordinate of the cubic spline.

Returns The second derivative of the cubic spline.

Return type np.ndarray

splinart.spline.splint module

Integration of a cubic spline.

splinart.spline.splint.**splint**(*xs*, *ys*, *y2s*, *x*, *y*)
Evaluate a sample on a cubic pline.

Parameters

- **xs** – The x coordinates of the cubic spline.
- **ys** – The y coordinates of the cubic spline.
- **y2s** – The second derivative of the cubic spline.
- **x** – The sample where to evaluation the cubic spline.
- **y** – The y coordinates of the sample.

See also:

splinart.spline.spline()

Module contents

Spline package

Submodules

splinart.color module

Define the default color of the output.

splinart.compute module

Material to update the output image using a cunbic spline equation.

splinart.compute.**update_img**(*img*, *path*, *xs_func*, *x=None*, *nrep=300*, *periodic=True*, *scale_color=0.005*, *color=(0.0*, *0.41568627450980394*, *0.6196078431372549*, *1.0)*, *scale_value=1e-05*)

Update the image using a cubic spline on a shape.

Parameters

- **img** (*np.ndarray*) – The output image.
- **path** (*np.ndarray*) – The y coordinate of the cubic spline if x is not None, the coordinates of the cubic spline if x is None.
- **x** (*np.ndarray*) – The x coordinates of the cubic spline if given. (the default value is None)
- **xs_func** (*function*) – The function that return the x coordinate of the sampling points where to compute the y coordinates given the spline equation.
- **nrep** (*int*) – Number of iteration (default is 300).
- **periodic** (*bool*) – Define if the first and last points of the path must be equal (default is True).
- **scale_color** (*float*) – Scale the given color (default is 0.005).
- **color** (*list(4)*) – Define the RGBA color to plot the spline.
- **scale_value** (*float*) – Rescale the random radius (default value is 0.00001).

See also:

update_path()

splinart.compute.**update_path**(*path*, *periodic=False*, *scale_value=1e-05*)

Update the path of the spline.

We move each point of the path by a random vector defined inside a circle where

- the center is the point of the path
- the radius is a random number between [-1, 1]

Parameters

- **path** (*np.ndarray*) – The y coordinate of the cubic spline.
- **periodic** (*bool*) – If True, the first and the last points of the path are the same (the default value is False).
- **scale_value** (*float*) – Rescale the random radius (default value is 0.00001).

splinart.draw module

Material to update the image with given points and save or plot this image.

splinart.draw.**draw_pixel**(*img*, *xs*, *ys*, *scale_color=0.0005*, *color=(0.0*, *0.41568627450980394*, *0.6196078431372549*, *1.0)*)

Add pixels on the image.

Parameters

- **img** (*np.ndarray*) – The image where we add pixels.
- **xs** (*np.ndarray*) – The x coordinate of the pixels to add.
- **ys** (*np.ndarray*) – The y coordinate of the pixels to add.
- **scale_color** (*float*) – Scale the given color (default is 0.0005).
- **color** (*list(4)*) – Define the RGBA color of the pixels.

splinart.draw.**save_img**(*img*, *path*, *filename*)
Save the image in a png file.

Parameters

- **img** (*np.ndarray*) – The image to save.
- **path** (*str*) – The save directory.
- **filename** (*str*) – The file name with the png extension.

splinart.draw.**show_img**(*img*)
Plot the image using matplotlib.

Parameters **img** (*np.ndarray*) – The image to save.

splinart.version module

Module contents

Splinart package

CHAPTER

FOUR

INDICES AND TABLES

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PYTHON MODULE INDEX

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